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Fault tolerance in noise-enhanced propagation

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As a mock-up of synaptic transmission between neurons, we revisit a problem that has recently risen the interest of several authors: the propagation of a low-frequency periodic signal through a chain of *one-way* coupled bistable oscillators, subject to uncorrelated additive noise. On a numerical study performed in the optimal range of noise intensity for which essentially undamped transmission along the chain has been reported, we focus on the outcome of *feeding with noise all the nodes but the central one*. A (moderate) critical value of the *coupling* between oscillators is found such that whereas below it the propagation can be considered to be interrupted at the ''dead neuron,'' it is *reestablished* above it. Thus, one of the distinguishing features of synaptic transmission, namely, a *fault-tolerant behavior* that enhances reliability at the expense of efficiency, arises here as an *emergent property* of the system.

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One of the striking effects of noise on nonlinear systems is the phenomenon called ''stochastic resonance'' (SR), consisting of a noise-assisted enhancement of coherence whereby power from the whole noise spectrum is pumped into a *single* mode coherent with the input signal [1,2]. Its potential relevance for neurophysiological processes was soon recognized and opened a vast research field [3,4]. The main point that triggered the first studies was the resemblance between interspike interval histograms of excited neurons and residence-time distributions of noise-driven bistable model systems where the two stable configurations correspond to the ''quiescent'' and ''firing'' states of the axon [3,5]. Through very delicate experiments that recorded the response of a *single* neuron submitted to a periodic stimulus under changing levels of noise, SR was indeed observed in neurons [6,7]. Also the spiky nature of neural response has been explicitly taken into consideration within the framework of SR [8,9].

In order to establish the significance of SR in *neural transmission* one must deal with a coupled *array* of bistable oscillators. Massive numerical simulations [10] and further analytical studies in continuum models [11,12] showed a *collective enhancement*, associated with synchronization, of the SR of a single element for a specific range in the noise intensity and the local coupling between ''stochastic resonators.'' The relevance of SR in the *transmission* along a line has been experimentally analyzed in Ref. [13] and discussed theoretically in Refs. [14,15].

The role of an uncorrelated noise to effectively improve signal transmission along a chain of *one-way* coupled bistable elements was established in Ref. [14]: in certain cases, essentially *undamped* signal transmission can be stimulated by the combined action of both injected signal and noise. In Ref. [15], a similar effect is found in one- and two-dimensional arrays of *two-way* coupled bistable oscillators: even for bilateral coupling, a moderate noise helps in eliciting synchronized oscillations and transmission along a line or network. Thus, the latter authors demonstrate the robustness of this phenomenon that they called *noise-enhanced propagation* (NEP), pointing out that their case is free from the possibility of a ''domino effect'' that is present in the setups of Refs. [13,14].

Two aspects of neural coupling cannot be overlooked: its *directionality* and its *fault tolerance*. Both have been certainly taken into account in all models of artificial neural networks [16] and in integrate-and-fire models (e.g., Ref. [9]). So it is our aim to explore the possibility of the second feature in a simple model that, seeking to mimic the synaptic transmission between neurons, accounts for the first one. The model we work with is by chance essentially the same as the one in Ref. [14] (of which we were unaware) but differently motivated: we regard it as an integrate-and-fire model, looked over times that are longer than the typical interspike periods. As in Ref. [14] we perform a numerical study of the unidirectional propagation of a small-amplitude, low-frequency periodic signal through a chain of asymmetrically coupled bistable oscillators, each submitted to an additive external noise (we chose our parameters to coincide with those in Ref. [1]). The ''active'' and ''quiescent'' neuronal states are *assumed* to be equally stable (by an *active* neuron we mean that a train of spikes is being transmitted along the axon, aiming to excite the next neuron along the line). The synaptic efficacy is represented by the coupling parameter ϵ' between one oscillator and the next one along the line. The Gaussian noise with which each oscillator is fed accounts for the random excitations of the environment to which the neuron responds. Whereas some of our results simply confirm the ones in Ref. [14] (for a large enough coupling we find an optimal level of noise for which the signal reaches the end with very low attenuation), we report in this Rapid Communication an additional and striking feature of this phenomenon: *the emergence in the system of a fault-tolerant behavior*. In fact, although generically external noise must be fed

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in *every* node for the signal to propagate along the chain, our numerical results show that *above a critical value of the coupling, an essentially undamped transmission is reestablished even when one of the nodes is deprived of noise*.

In the absence of noise, the (subthreshold) signal is rapidly damped. An optimal amount of noise counterbalances this effect, greatly enhancing the transmission. The coupling between oscillators along the line helps in turn to overcome the random perturbation produced by the external sources of noise. In Refs. [14,15] an optimal range in the noise level was found for which a good transmission is achieved, even for ϵ' not too high. We find that if the noise intensity exceeds this optimal range a good transmission can still be achieved, but at the expense of a very large coupling ϵ' (that grows exponentially with D). The main novelty is however that the process is found to be *fault-tolerant*, i.e., that it is *robust against accidental interruptions of the chain of oscillators*. This is checked by depriving of noise one of the neurons at the center of the chain: if the coupling exceeds a (moderate) critical value, the signal nevertheless shows the ability to ''jump'' over the ''dead'' oscillator and to reach the end of the chain with a negligible additional attenuation.

We consider a one-dimensional chain of nonlinear oscillators operating in an overdamped regime. Let x_n be the amplitude of the nth oscillator, $n=0,1\ldots N$, and $V(x_n)$ be the usual double-well potential with minima of depth U_0 located at $x=\pm c$ [1]:

$$V(x)=-U_0\left(\frac{x}{c}\right)^2\left[2-\left(\frac{x}{c}\right)^2\right]. \tag{1}$$

Each oscillator is submitted to an external driving force F_n^{dr}, which is proportional to x_{n-1} for $n>1$ and to a periodic signal of frequency ω_{ex} for $n=1$,

$$F_n^{dr}=\begin{cases}\epsilon\cos(\omega_{ex}t) & \text{for } n=1\\ \dfrac{\epsilon'}{c}x_{n-1}(t) & \text{for } n>1.\end{cases} \tag{2}$$

Having chosen the units so that the damping coefficient γ for the intrawell dynamics is 1 and all amplitudes are measured in units of c, the two coupling constants ϵ and ϵ' in Eq. (3) have the same dimensions as $\dot{x}$. We have omitted any delay in the coupling because it introduces no relevant conceptual ingredient into the model. In addition, we assume that all the oscillators are in the presence of uncorrelated Gaussian noise sources $G_n(t)$ with unit power-spectral density, coupled with strength $\sqrt{D}$. The set of coupled equations of motion is therefore

$$\dot{x}_n+\frac{dV}{dx_n}=F_n^{dr}+\sqrt{D}G_n(t). \tag{3}$$

A necessary condition for this transmission line to work is that the first oscillator be near the regime of stochastic resonance. If the amplitude D of the perturbing noise is properly tuned to the height of the barrier $D_{SR}\sim U_0/2$, the transition rate between wells matches ω_{ex} and the first oscillator performs large-amplitude oscillations that are statistically periodic but anharmonic and noisy. An enhanced value of the signal-to-noise ratio (SNR) R thus appears in the power spectrum of the first oscillator at the same frequency of the external periodic driving excitation (the SNR is defined as the ratio between the values of the output power spectrum with and without signal and is usually expressed in dB). The coupled equations (3) reproduce to a large extent the same situation for the next oscillator along the line, thus making possible the propagation of the periodic excitation along the transmission line. However, some further tuning of D has to be performed because each oscillator is driven by *two* sources of noise: the external one $G_n(t)$ with amplitude D and the background noise that comes superimposed to the periodic signal of the preceding oscillator. Since the latter is fed into the next oscillator through the coupling ϵ', one expects the propagation of the signal along the line to be largely governed by ϵ'.

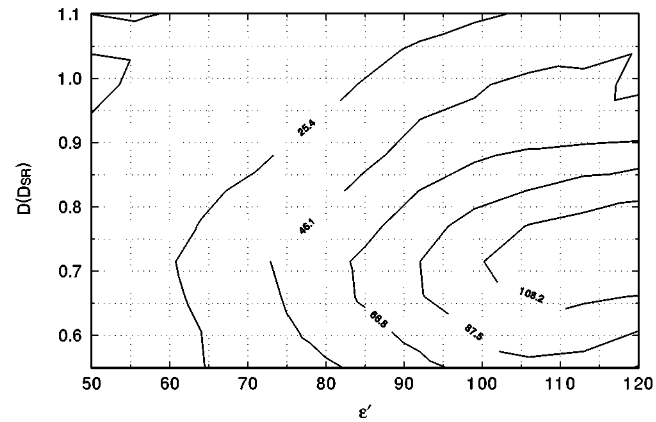

FIG. 1. Contour plot of the decay length λ as a function of the coupling ϵ' and the noise amplitude D (in units of U_0). The value of λ is estimated in each case through a linear regression of Δ_n as a function of n in a line of 20 oscillator links. From left to right, the values of λ are 20.6, 36.6, 52.5, 68.4, 84.4, 100.3, and 116.3.

We have integrated Eqs. (3) adopting the following values [1]: $U_0=256$, $c=4\sqrt{2}$, $\omega_{ex}=0.39\pi$, and have also averaged the power spectra over a suitable ensemble of random initial conditions in order to hinder the background fluctuations. The main free parameters in our calculation are D (which we choose to express in units of U_0) and ϵ'. To provide a quantitative estimate of the transmission we assume an exponential decay in the SNR: $R(n)\sim R(0)\exp(-n/\lambda)$, where λ is a decay length that provides an estimate of the effectiveness of the transmission line. Its value can be estimated through the difference $\Delta_n=R(1)-R(n)$, where the values of R are expressed in dB [17].

Figure 1 shows the contour plot of λ for different values of D and $\epsilon'\geqslant 50$ (for $\epsilon'<50$ the chain is fragile against noise and a large attenuation always occurs). The best performance is seen to occur for $D\approx 0.7U_0$ and $\epsilon'\geqslant 60$: the line displays a transmission regime with an attenuation of less than 1 dB. The curve R versus D (not shown) becomes more narrowly peaked as one travels further along the line. The ''resonant'' noise intensity can thus be defined with better precision than for a single oscillator.

As seen in Fig. 1, it is possible to overcome the hindrance that a stronger noise exerts to transmission by paying the overhead of a larger value of the coupling. An increasing value of ϵ' transforms progressively the set of loosely linked oscillators into a chain of rigidly coupled ones. The effect of

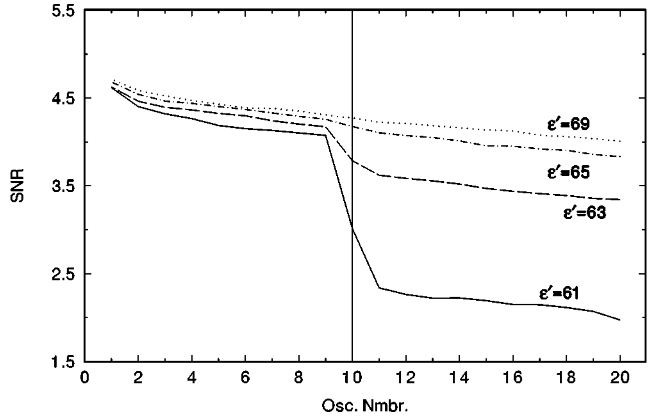

FIG. 2. Plot of the signal-to-noise ratio R along a line of 20 bistable oscillators, showing the emergence of a fault-tolerant behavior for $\epsilon' \geqslant 63$. The noise amplitude is $D = 0.8U_0$ for all the oscillators except at the tenth link (where the vertical line is placed), for which $D = 0$. The values of the coupling ϵ' are 61 (solid line), 63 (dashed line), 65 (dot-dashed line), and 69 (dotted line).

the transition towards this regime is, however, not uniform: there is a *best value* of the noise intensity $D \sim 0.7U_0$ that helps reaching the regime of undamped transmission for lower values of ϵ' (a kind of ''stochastic tunneling'') [14,15]. Below this value no transmission takes place because not enough energy is pumped into the coherent oscillation of each link of the line. Above it, the value of ϵ' required to overcome the increasing noise grows rather fast with D. In order to see this we have studied the dependence on D of the value of ϵ'_{cr} for which Δ_{20} has some prescribed value. We find from our numerical experiments that $\log \epsilon'_{cr}$ scales approximately as $2D/3$.

The transmission line that we consider is able to transmit a signal *because* noise is fed to its nodes. Instead of disturbing, a moderate amount of noise *helps* the transmission. This can be seen by interrupting the chain through not feeding noise to one of the links of the line. In Fig. 2 we show the SNR along the line for $D = 0.8U_0$ and several values of ϵ'. The line has 20 oscillators, and the tenth oscillator is ''dead.'' The transmission can be seen to easily overcome the presence of a missing link (showing no appreciable damage) provided that ϵ' is above a threshold value. For the settings that we have chosen, the latter turns out to be $\epsilon'_{th} \simeq 63$. This value is quite sharply defined (i.e., values of ϵ' outside a rather narrow window centered in ϵ'_{th} change drastically the attenuation) and is *noticeably smaller* than the one required to restore propagation in the chain free of defect for $D > 0.8U_0$. Our numerical results show that ϵ'_{th} is a slowly varying function of the noise amplitude D. Fault tolerance is therefore to be attributed to the *coupling* of consecutive oscillators along the line, magnified by the external noise source as in the stochastic resonance of an isolated bistable system.

In order to have a global picture of the combined effect of D and ϵ' we analyze the spatiotemporal correlations of the bistable oscillators in Fig. 3. We use the same convention as Ref. [10], that so clearly displays these effects. A black (white) pixel indicates that the oscillator is in the left (right) well. Since the distance along the line is measured along the y axis of each graph, an undisturbed transmission is reflected by patterns of vertical stripes. The pattern is disturbed in two ways: on one hand, the *pumping* of noise into each oscillator causes accidental black and white pixels to appear within a stripe of the opposite color (this is seen to happen more and more frequently as D is increased in succesive graphs towards the right); on the other, the *absence* of noise at the tenth oscillator is a hindrance for the statistically periodic switching downstream.

The plots in the lower left corner show a good transmission only up to the tenth oscillator. When the noise amplitude is low, the following links of the line remain undisturbed in either of the two wells for a much longer period. For large values of D the oscillators jump from one side to the other without any synchronization (bottom line of graphs). The transmission is restored in the plots of the upper left corner. The value of ϵ' has been increased beyond $\epsilon'_{th} \simeq 63$ and consequently the discontinuity at $n = 10$ has completely disappeared. The division line at $n = 10$ is clearly visible throughout the range $0.65U_0 \leqslant D \leqslant 1.1U_0$ and $\epsilon' \leqslant 63$. For larger D the value of ϵ'_{th} required to have a perfect transmission is slightly larger (see, e.g., the graph corresponding to $\epsilon' = 64$ and $D = 0.95U_0$ or $1.1U_0$). However, at that stage transmission is in a severe compromise even within the segments of the line that have no interruptions. This can be seen by the fact that the pattern of vertical stripes has almost disappeared in the graph of the upper right corner.

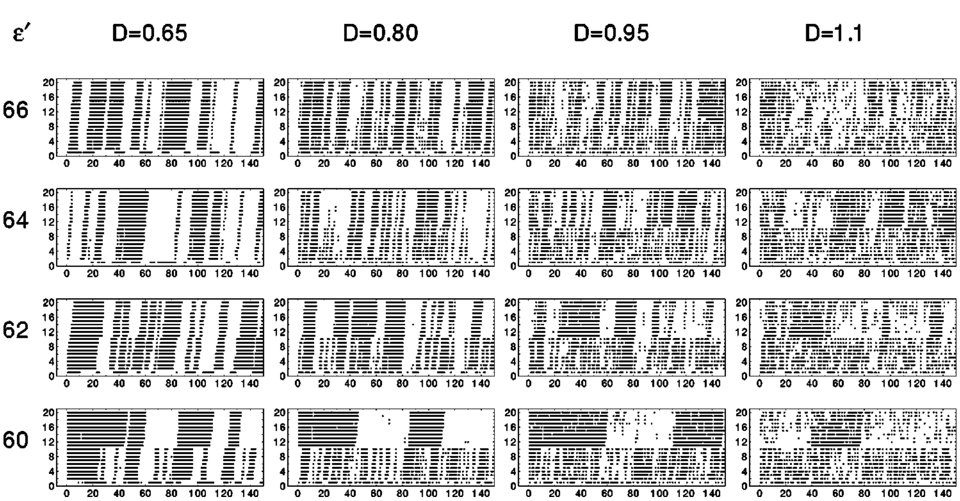

FIG. 3. Spatiotemporal correlations in a chain of 20 oscillators with its tenth link deprived of noise. For each plot, time runs along the x axis (each unit corresponds to 200 integration steps), while oscillator number runs along the y axis. The noise amplitude D is displayed below each column (in units of U_0) and the coupling strength ϵ' at the left of each row.

We have considered a mock-up of neural transmission through synaptic links. Our simulations involve two free parameters: the intensity of the (asymmetric) coupling along the line ϵ' and the amplitude D of the uncorrelated external noise sources. The former plays the role of the synaptic efficacy and the latter represents the random response of the neurons to environmental stimuli. A better transmission is always achieved for larger values of ϵ'. A kind of ''stochastic tunneling'' is found for D between $0.7U_0$ and $0.8U_0$, a region in which a good transmission is achieved for much lower values of ϵ'. If D is too small not enough energy is pumped into a coherent motion in each oscillator and the signal is rapidly damped; if it is too large, random fluctuations cause incoherent transitions between the left and right wells of the bistable oscillators and no transmission can take place. The region of optimal values of D is independent of ϵ', since the ridge displayed in Fig. 1 at $D \sim 0.75U_0$ does not bend in any direction. The parameter ϵ' largely governs the damping along the transmission line. If ϵ' is too low, the periodic signal is not fed into the next oscillator of the chain and the transmission is damped exponentially. Instead, the harmful effects of an excessive noise can be overcome for ϵ' large enough and the whole chain is able to transmit the periodic signal, behaving like a strongly coupled (''rigid'') system.

The relevance of the external noise sources in sustaining the transmission is dramatically displayed by the fact that transmission can be interrupted by depriving of noise one of the oscillators of the chain. However, the interruption can be overcome if $\epsilon' > \epsilon'_{th}$, thus providing some degree of fault tolerance to the transmission network. The value of ϵ'_{th} is sharply defined and lies far from the strong-coupling limit for the corresponding level of noise.

Fault tolerance is a well known feature of neural transmission. The existence of a threshold value ϵ'_{th} together with the adaptive (perhaps Hebbian) change of synaptic efficacies opens up the possibility to model the progressive stabilization of transmission patterns of neural excitations. The results in Refs. [10] and [15] suggest that the present one can be extended to a highly interconnected neural network. The extension to a layered one is being undertaken, as well as a check of its robustness against disorder in ϵ'.

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